LETTER TO THE EDITOR

A DIFFERENTIAL METHOD OF INVESTIGATING RADIATIVE HEAT TRANSFER
(apropos the article by P. K. Konakov in: IFZh [Journal of Engineering Physics], No. 3, 1965)

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In a letter to the editors of the Journal of Engineering Physics, Konakov [1] made critical comments on our papers [2, 3] and on the work of Hottel [7, 8], with the object of defending his own conception of radiative heat transfer [4, 5, 6], which has been subjected to criticism on a number of occasions [2, 3, 7, 8, 9].

Firstly, Konakov tries to dispute the correctness of the formulation of boundary conditions for the diffusion method, which is reviewed in [2, 3]. These boundary conditions relate the radiative flux q at the boundary between wall and medium with the wall temperature T_W (at the boundary with the medium) and the volume density of radiative energy U_f in the medium, at the boundary with the wall. Assuming an isotropic intensity distribution in the incident and effective fluxes, they may be written as follows:

$$q = \frac{(cU)_f/4 - \sigma_0 T_w^4}{1/A_w - 1/2}. \qquad (1)$$

These equations were first formulated at the Krzhizhanovskii Power Engineering Institute in 1940 [10], and subsequently found wide application in investigations of radiative heat transfer [11–14], and a monograph (p. 107 [5]), one of the authors of which is Konakov himself. However, he regards that manner of writing the boundary conditions as incorrect, and to show this he cites the fact that if the equality

$$(cU)_f/4 = \sigma_0 T_w^4, \qquad (2)$$

is assumed, then, according to (1), the radiative flux at the medium-wall boundary will be zero (q = 0), whereas in the layer of the medium itself it will have quite a definite value. Then the author of the letter erroneously assumes that equality (2) must follow from the condition of local radiative equilibrium in the medium

$$cU/4 = \sigma_0 T_f^4, \qquad (3)$$

which was assumed in solving the problem of radiative transfer in a plane layer of the medium. In reality, it is quite clear that (2), which would indicate equality of the temperatures of the wall and the medium at the boundary with the wall, in no way follows from the condition of local radiative equilibrium (3) inside the medium. It is evident that (2) is satisfied only for thermodynamic equilibrium in the system, when the wall temperatures are equal to that of the medium and there is no resultant heat transfer.

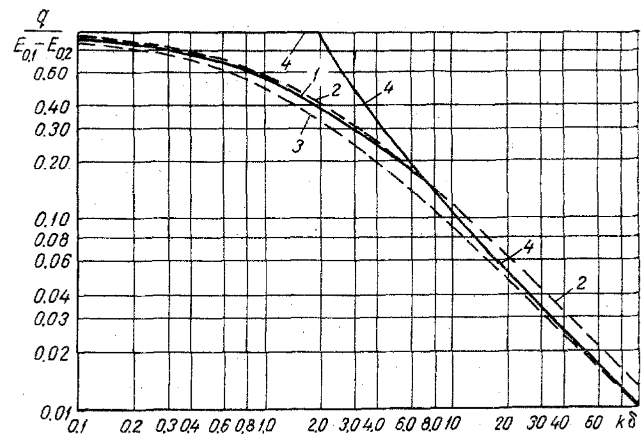

Fig. Dependence of dimensionless flux $q/(E_{0.1} - E_{0.2})$ in a plane layer of absorbing medium on optical thickness kδ, as calculated by various methods: 1) from numerical solution of the integral equation as performed by Hottel [7, 8]; 2) from the diffusion method formula [2, 3, 13, 14]; 3) from the differential-difference method formula described by Schuster and Schwarzschild [2, 3]; 4) from the Konakov solution [5, 6].

Also apparently without proof is Konakov's assertion that his formulation of the boundary conditions [4] supposedly uses Bouguer's law, whereas he ignores the interaction of radiation with the medium at a distance equal to a photon mean free path ($k\delta \le 1$). It is well known, however, that the attenuation of a ray over this length is equal to 63.2%. The author of the letter evidently conveniently borrows the assumption from the kinetic theory of molecules, where with system dimensions much greater than the molecular mean free path, interaction between molecules is neglected at distance less than the mean free path. This analogy is too crude for actual values of the optical thickness of the medium (kδ), and for this reason, as has been shown in [2, 3, 7, 8, 9], its adoption leads to appreciable errors. Konakov therefore formally achieved the limiting transition to the Christiansen formula for spherical and cylindrical layers of medium, because, for layers of optical thickness less than $k\delta < 2$, he simply excluded attenuating media from consideration. On this basis it cannot be asserted that the boundary conditions (1) are incorrect, while Konakov's formulation [4] of the boundary conditions is supposedly more correct. It can be clearly seen from (pp. 77–78 [3]) that the diffusion formula does not give the limiting transition to the Christiansen formula, because the last term of (14) is usually neglected in [3].

It is also asserted in the letter that the radiative diffusion coefficient is equal to c/4k for any optical layer thickness, and not c/3k, as follows from the work of Rosseland [15], Genzel [16], Deissler [14], and others. In support of this assertion, Konakov cited the quantity $D = c/4k$ recommended by Shorin [12] for a plane layer of weakly absorbing medium, although in the last edition of this same book [13], this author assumes a value $D = c/3k$ in discussing the problem of radiative transfer in a plane layer of attenuating medium. The fact of the matter is that in a plane layer of medium of very small optical thickness ($k\delta \ll 1$) and with diffusely radiating walls, the diffusion coefficient approximates to the value c/4k. However, when the optical thicknesses of a plane layer are considerable ($k\delta \geqslant 6$), its value is $c/3k = cl/3$, as exact theory indicates (in a similar manner to the molecular diffusion coefficient in an isotropic medium). It is precisely the choice of the diffusion coefficient equal to c/4k instead of c/3k that explains why the Konakov formula at large $k\delta$ gives results 25% less than the exact values. This is clearly shown in the figure, where a comparison is made of solutions obtained by different methods for the problem of radiation transfer in a plane layer. In spite of the evidence, the author of the letter asserts that his solution coincides with Hottel's curve [7, 8] after values $k\delta \geq 6$, although in fact his curve only intersects the Hottel curve 1 in the region $k\delta \approx 8.0$, and subsequently gradually approaches curve 3, obtained by the Schuster-Schwarzschild method, which lies 25% below the exact values. When $k\delta = 2$, the assumption of boundary conditions according to Konakov [5, 6], as may be seen from the graph, gives an error in excess of 150%.

Finally, the author of the letter attempts to cast doubt on the clarity of the mathematical formulation of the problem solved by Hottel and his colleagues [7, 8]. He has no basis for this, since the solution in question was based on a sufficiently rigorous and exact integral radiation equation in a plane layer bounded by black walls. Since the Hottel solution was obtained on the basis of a zonal approximation with a large number of zones, no doubt arises in the range of $k\delta$ values investigated as to the adequate accuracy of the results obtained. This is confirmed by the good agreement between Hottel's results and those obtained by the tensor and differential diffusion methods [2, 9]. *

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Editors' Note: This letter is a reply to the remarks of P. K. Konakov, published in Journal of Engineering Physics, Vol. 8, No. 3, 1965. The Editors consider that further discussion of this question would be inappropriate, and suggest that those participating consider the points raised at one of the seminars on methods of calculating radiative heat transfer.

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*After the present letter had been sent to the editor, a Russian translation appeared of an article by Chislet and Baldwin ("Raketnaya tekhnika i kosmonavtika," no. 12, 1964), in which the results of our paper [2] are examined. Chislet and Baldwin point out (p. 76) that the equation obtained by us is in good agreement with the solutions of other authors.